

Queues



Customers have complained to a newsagent about the length of the queue in his shop during the busy period between 8.15 am and 8.45 am when people stop to buy a newspaper and other items on their way to work.

Situations such as this can be investigated with simulations carried out using random numbers. There are 10 equally likely random digits, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, each having a probability of occurring of 0.1. Find out how to generate random numbers on your calculator and look carefully at the numbers produced. It may surprise you to see that sometimes a particular digit does not appear for quite a while and that repeat digits are quite common.

To design a simulation for the newsagent's situation we need to have some information about how often customers join the queue and how long it takes to serve them. Suppose that the newsagent has observed that on average customers join the queue at a rate of one per minute and that the time taken to serve them (to the nearest half minute) varies as shown in the table.

Time taken to serve (min)	0.5	1	1.5	2	2.5	3	Note the total probability is 1
Probability	0.1	0.3	0.2	0.2	0.1	0.1	

Random digits must be assigned to each of these times according to their probabilities. Each time having a probability of 0.1 should be assigned one random digit, the times that have a probability of 0.2 should be assigned two random digits and the time that has a probability of 0.3 should be assigned three random digits. This can be done as shown in the table below:

Time taken to serve (min)	0.5	1	1.5	2	2.5	3
Probability	0.1	0.3	0.2	0.2	0.1	0.1
Random Digits	0	1, 2, 3	4, 5	6, 7	8	9

When designing a simulation, it is useful to organise the results in a table. A suitable table for this situation has been started below. The first column shows consecutive customers. The second column shows the time at which each customer joined the queue, assuming that they arrive regularly at a rate of one customer per minute.

Assuming that there is no queue when the first customer arrives, he or she will be served immediately at 8.15 am. The variable in this situation is the time taken to serve customers. Random numbers from a calculator are used to simulate serving times, using the table above. The time when the newsagent finishes serving the customer can then be found. The last two columns show what is important to customers – the length of the queue and how long they have to wait. Carefully check all of the entries in the table.

Customer Number	Arrival Time	Serve starts at	Random Number	Serving Time (min)	Serve ends at	Wait Time (min)	Queue length at time of arrival
1	8:15	8:15:00	0	0.5	8:15:30	0	0
2	8:16	8:16:00	9	3	8:19:00	0	0
3	8:17	8:19:00	5	1.5	8:20:30	2	0
4	8:18	8:20:30	2	1	8:21:30	2.5	1
5	8:19	8:21:30	9	3	8:24:30	2.5	1
6	8:20	8:24:30	1	1	8:25:30	4.5	2



The table has been extended below to show the other customers who arrive before 8.45 am. Use the given random numbers to complete the table.

Customer Number	Arrival Time	Serve starts at	Random Number	Serving Time (min)	Serve ends at	Wait Time (min)	Queue length at time of arrival
1	8:15	8:15:00	0	0.5	8:15:30	0	0
2	8:16	8:16:00	9	3	8:19:00	0	0
3	8:17	8:19:00	5	1.5	8:20:30	2	0
4	8:18	8:20:30	2	1	8:21:30	2.5	1
5	8:19	8:21:30	9	3	8:24:30	2.5	1
6	8:20	8:24:30	1	1	8:25:30	4.5	2
7	8:21	8:25:30	9				
8	8:22		0				
9	8:23		8				
10	8:24		4				
11	8:25		6				
12	8:26		6				
13	8:27		5				
14	8:28		9				
15	8:29		7				
16	8:30		9				
17	8:31		1				
18	8:32		6				
19	8:33		7				
20	8:34		6				
21	8:35		8				
22	8:36		0				
23	8:37		3				
24	8:38		9				
25	8:39		1				
26	8:40		3				
27	8:41		9				
28	8:42		0				
29	8:43		9				
30	8:44		2				

What is the longest time you think that it is acceptable to wait in a newsagent's queue? Do you think that the simulation indicates that the customers' complaints are justified?

Graphs can be used to illustrate the results of the simulation. Customers are likely to be interested in the time they have to wait and the length of the queue. There are various ways these can be shown on a graph.

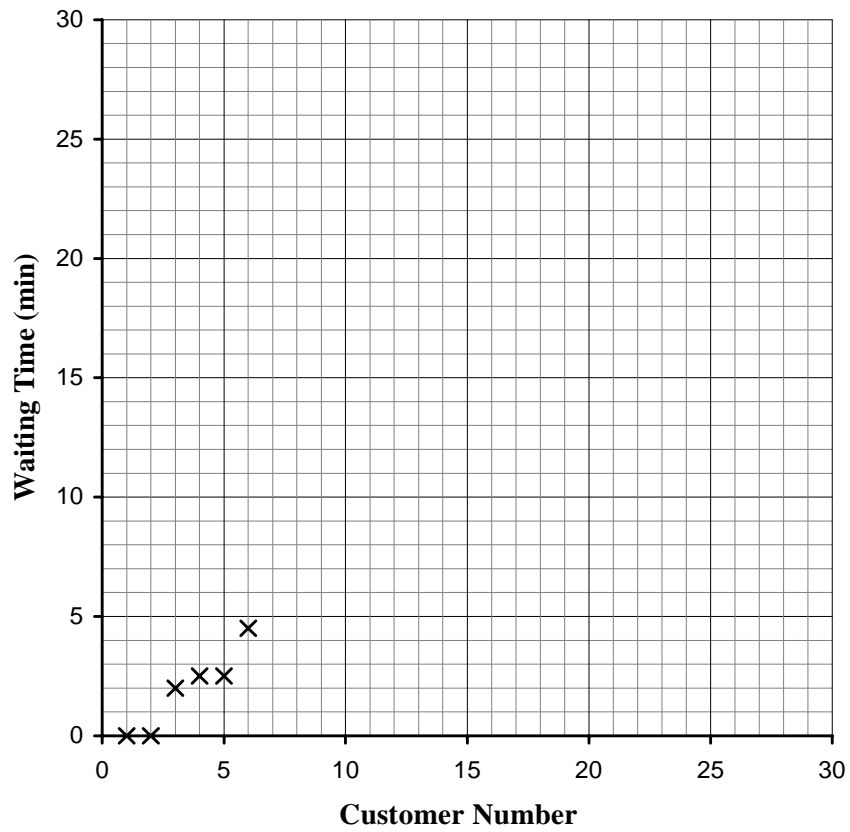
The first graph on the following page shows waiting time plotted against customer number. It would also be useful to use time on the x axis.

The second graph shows the queue length plotted against time.

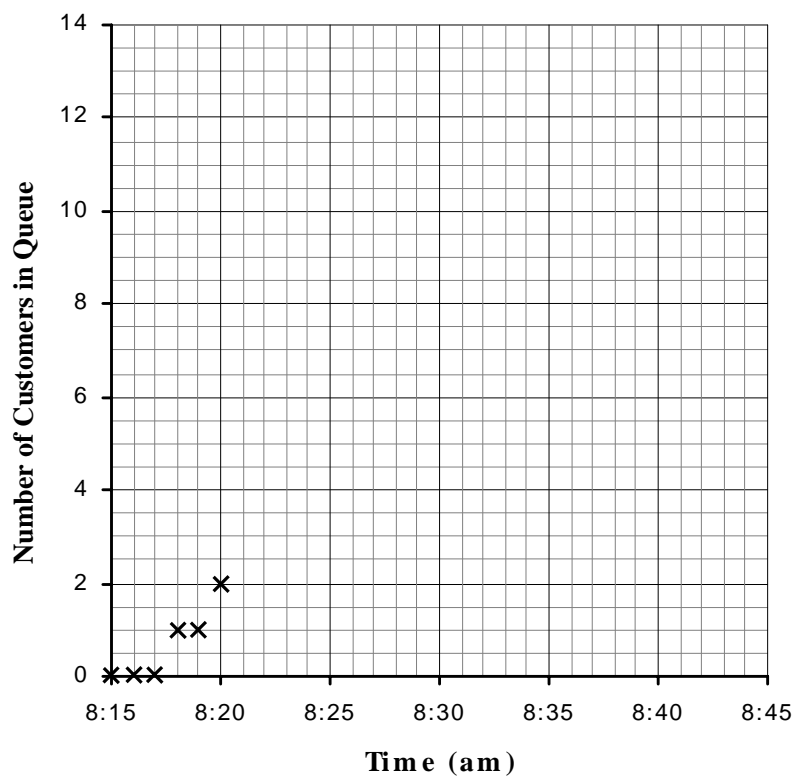
Use the values in your table to complete these graphs.



Customer waiting times



Queue length between 8.15am and 8.45am



Any simulation should be tried out a number of times to see whether the results it gives are consistent. The first run may have given an unusual result.

Complete the table and graphs below to show the results of another run of the simulation. This time use your calculator to find random digits for the fourth column.

Customer Number	Arrival Time	Serve starts at	Random Number	Serving Time (min)	Serve ends at	Wait Time (min)	Queue length at time of arrival
1	8:15						
2	8:16						
3	8:17						
4	8:18						
5	8:19						
6	8:20						
7	8:21						
8	8:22						
9	8:23						
10	8:24						
11	8:25						
12	8:26						
13	8:27						
14	8:28						
15	8:29						
16	8:30						
17	8:31						
18	8:32						
19	8:33						
20	8:34						
21	8:35						
22	8:36						
23	8:37						
24	8:38						
25	8:39						
26	8:40						
27	8:41						
28	8:42						
29	8:43						
30	8:44						

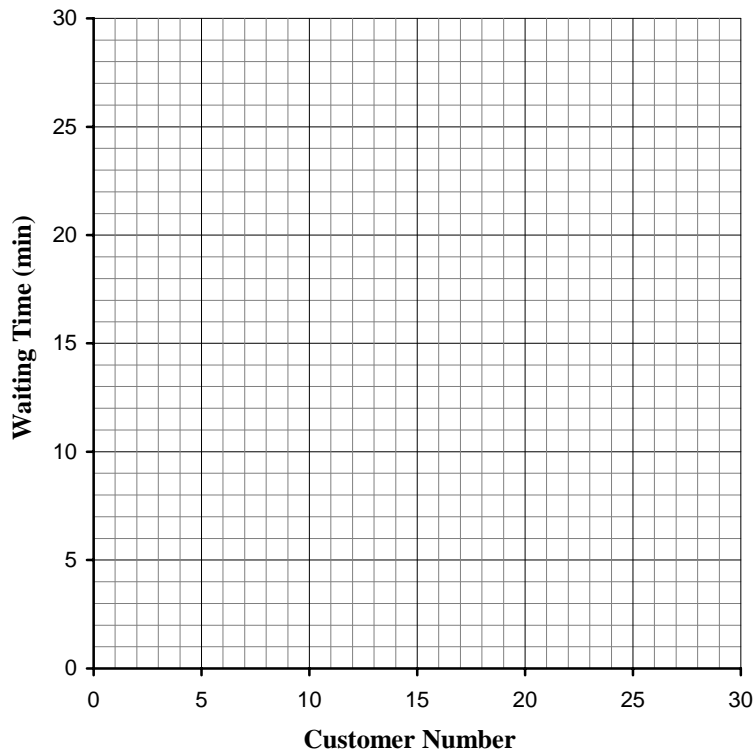
Compare the results of this run with the first run (and with other runs of the simulation if possible). Write a brief summary of your findings.

In practice customers are not likely to join the queue regularly every minute. Also customers are unlikely to join the queue if it looks like they will have to wait a long time. If the simulation is found to be a poor reflection of the real situation, extra sets of random numbers can be used to simulate other variables such as the time between consecutive arrivals and the time customers are prepared to wait in an effort to give more realistic results.

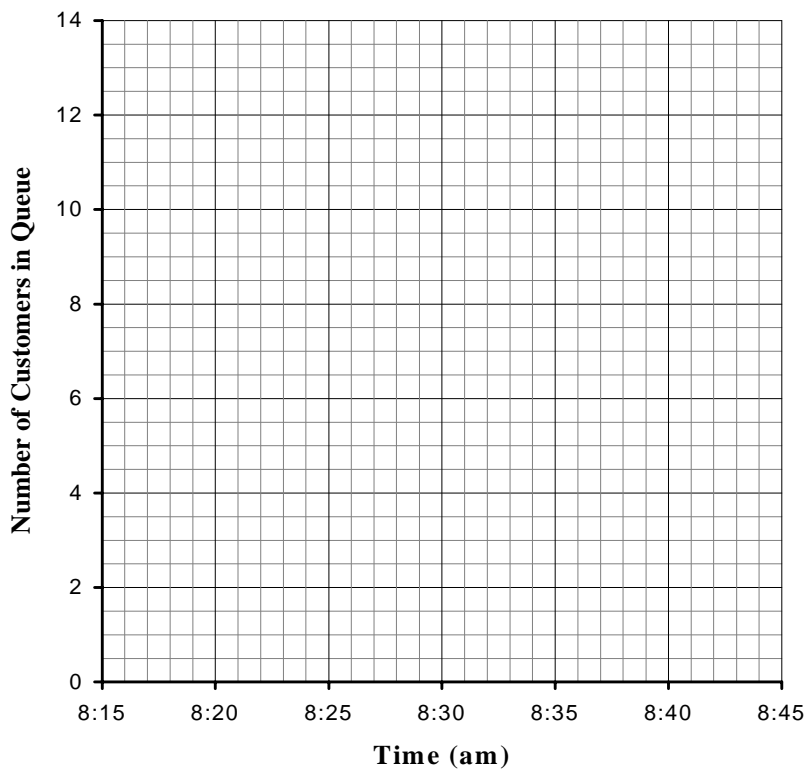
The newsagent might consider employing an assistant to help him during the busy period. Before doing this he could use another simulation to indicate whether this is likely to solve the problem and whether the assistant would be fully occupied during this time.



Customer waiting times



Queue length between 8.15am and 8.45am



Teacher Notes

Unit Advanced Level, Applying mathematics

Notes

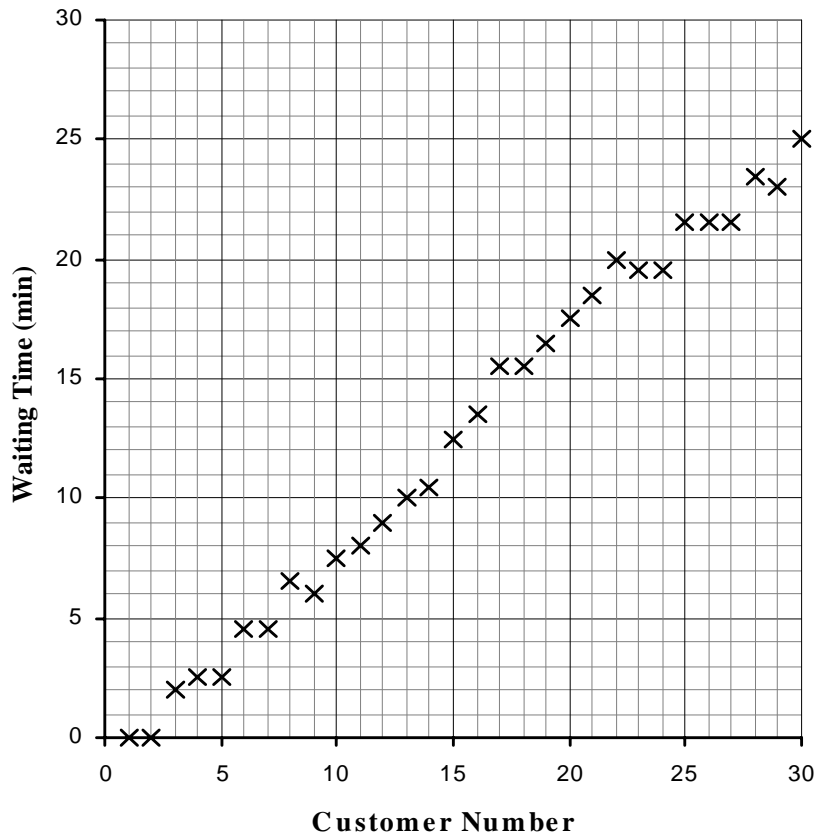
This activity involves the use of random numbers to carry out a simulation. Students will need to be familiar with probability. They will also need to have a calculator that generates random numbers. On graphic calculators the random number generator is usually listed in the probability menu as RAN# or RAND#. All answers can be filled in on the activity sheets. It would be useful to pool students' results to show the variation between runs of the simulation. Class discussion could be used to share ideas and summarise the important points. The work could be extended to include a second set of random numbers to simulate different arrival times. The employment of an assistant could also be investigated.

Answers NB The person being served and the person joining the queue have not been counted as part of the queue.

Customer Number	Arrival Time	Serve starts at	Random Number	Serving Time (min)	Serve ends at	Wait Time (min)	Queue length at time of arrival
1	8:15	8:15:00	0	0.5	8:15:30	0	0
2	8:16	8:16:00	9	3	8:19:00	0	0
3	8:17	8:19:00	5	1.5	8:20:30	2	0
4	8:18	8:20:30	2	1	8:21:30	2.5	1
5	8:19	8:21:30	9	3	8:24:30	2.5	1
6	8:20	8:24:30	1	1	8:25:30	4.5	2
7	8:21	8:25:30	9	3	8:28:30	4.5	2
8	8:22	8:28:30	0	0.5	8:29:00	6.5	2
9	8:23	8:29:00	8	2.5	8:31:30	6	3
10	8:24	8:31:30	4	1.5	8:33:00	7.5	4
11	8:25	8:33:00	6	2	8:35:00	8	4
12	8:26	8:35:00	6	2	8:37:00	9	4
13	8:27	8:37:00	5	1.5	8:38:30	10	5
14	8:28	8:38:30	9	3	8:41:30	10.5	6
15	8:29	8:41:30	7	2	8:43:30	12.5	5
16	8:30	8:43:30	9	3	8:46:30	13.5	6
17	8:31	8:46:30	1	1	8:47:30	15.5	7
18	8:32	8:47:30	6	2	8:49:30	15.5	7
19	8:33	8:49:30	7	2	8:51:30	16.5	7
20	8:34	8:51:30	6	2	8:53:30	17.5	8
21	8:35	8:53:30	8	2.5	8:56:00	18.5	8
22	8:36	8:56:00	0	0.5	8:56:30	20	9
23	8:37	8:56:30	3	1	8:57:30	19.5	9
24	8:38	8:57:30	9	3	9:00:30	19.5	10
25	8:39	9:00:30	1	1	9:01:30	21.5	10
26	8:40	9:01:30	3	1	9:02:30	21.5	11
27	8:41	9:02:30	9	3	9:05:30	21.5	12
28	8:42	9:05:30	0	0.5	9:06:00	23.5	12
29	8:43	9:06:00	9	3	9:09:00	23	13
30	8:44	9:09:00	2	1	9:10:00	25	13



Customer waiting times



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